Thruster-Augmented Active Control of a Tethered Subsatellite System During Its Retrieval

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The paper considers control of the rotational motion as well as longitudinal and transverse vibrations of a tethered subsatellite system during its retrieval to the shuttle. Control using a set of thrusters alone is studied first; then, a length rate control law augmented by thrusters is examined. The functional forms of the thrust components applicable to both cases are determined by analyzing simplified equations of motion. These are validated by computer simulation of the original equations. The schemes considered appear to be fairly effective in arresting the growth of rotations as well as vibrations while maintaining a nonzero tension in the tether. It is recommended that the exponential retrieval be replaced by uniform retrieval rate toward the end of retrieval.

Nomenclature

A = area of cross section of the tether
A_i, B_i = coefficients of ϕ_i in the expansion of u and w, respectively, Eqs. (1a) and (1b)
c = retrieval constant
c = cos, as abbreviated in Eqs. (A1-A8)
$\tilde{c}$ = nondimensional retrieval constant, $\tilde{c} = c/\omega$; $L(\theta) = L(0)\exp(\tilde{c}\theta)$, $\tilde{c} < 0$ for exponential retrieval
C_i = coefficient of ψ_i in the expansion of v, Eq. (1c)
C_{is} = steady-state value of C_i
e = eccentricity of the orbit
E = Young's modulus of the tether material
i = inclination of the orbit to the equatorial plane
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ = unit vectors along x, y, z axes, respectively
k = a constant, $\sqrt{2}/\pi$
$K_i, \overline{K}_i$ = gains, Eqs. (13), (15-17); $i \equiv \alpha, \gamma, A_1, A_2, B_1, B_2, C$
L, L_i = unstretched length of the tether at any instant and at the beginning of retrieval, respectively
L_T = length below which the thrusters fire additionally to maintain a minimum tension in the tether
L_1, L_2 = used in mixed control strategy; control changes from length rate control to thruster control at $L = L_1$; exponential retrieval changes to constant velocity retrieval below $L = L_2$
m_s = mass of the subsatellite
P_i, R_i = nondimensional environmental and control generalized forces, respectively, $i \equiv \alpha, \gamma, A_i, B_i, C_i$
Q_i = generalized forces, $i \equiv \alpha, \gamma, A_i, B_i, C_i$
$\boldsymbol{R}_s$ = position vector of the subsatellite relative to the center of mass of the system
s = sin as abbreviated in Eqs. (A1-A8)
T_α, T_c, T_γ = components of the thrust vector along local z, y, x directions, respectively
$\tilde{T}_\gamma, \tilde{T}_c, \tilde{T}_\alpha$ = nondimensional values of the thrust components, $\tilde{T}_\gamma = T_\gamma / m_s L \dot{\theta}^2$, etc.
tg = tangent, as abbreviated in Eqs. (A1-A8)
u = out-of-plane transverse displacement of an arbitrary point on the tether
v = longitudinal displacement of an arbitrary point on the tether
w = in-plane transverse displacement of an arbitrary point on the tether
x, y, z = tether-based local coordinate system; y is along the line joining the two points of attachment of the tether
x_o, y_o, z_o = orbital coordinate system; y_o axis is along the radial vector and x_o axis the orbit normal
α = in-plane rotation of the tether (pitch)
γ = out-of-plane rotation of the tether (roll)
ϵ = strain in the tether at any arbitrary point
η = nondimensional length, $\eta = \ln(L/L_i)$
θ = true anomaly
ν = mass ratio, $\rho_t L/m_s$
ρ_t = mass per unit length of the tether
σ = tension at any arbitrary element of the tether
ϕ_i, ψ_i = admissible functions used in the expansions of u, v, w, Eq. (1)
ω = mean orbital rotational velocity
ω_c = $(EA/m_s L\dot{\theta}^2)^{\frac{1}{2}}$
Ω = $(EA/\rho_t L^2 \dot{\theta}^2)^{\frac{1}{2}}$

Superscripts
$\dot{(\ )}$ = differentiation with respect to t
$(\ )', (\ )''$ = differentiation with respect to θ

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Introduction

THE possibility of using tethers in space has generated considerable interest. In particular, tethered subsatellite systems deployed from the shuttle have been proposed for a multitude of uses. Unfortunately, the rotational motion of such a system, as well as the longitudinal and transverse vibrations of the tether, grows with time during the retrieval phase. The problem of controlling this unstable general dynamics has not been solved completely yet, even though a significant effort has been concentrated on it.[1] As the length of the tether reduces to a small value, the equilibrium tension in the tether due to the gravity gradient approaches zero, and during a dynamical situation the tether may become slack. Thus a tension control law (for example, the one suggested by Baker et al.[2]) or any modification of that such as a length rate law[3] becomes ineffective. To alleviate this difficulty, Banerjee and Kane[4] have proposed using a set of thrusters (in addition to a torque control law) to control the retrieval dynamics. The results presented in their paper are quite promising; however, the transverse vibrations of the tether were neglected in their dynamical model although the longitudinal vibrations were taken into account. The two types of vibrations are strongly coupled (as will be seen later), specifically when the tension in the tether is small.[5] The objective of this paper is to present thruster-augmented control schemes that are effective in the presence of both longitudinal and transverse vibrations.

A nonlinear dynamical model[5] developed earlier by the authors to study the vibrations is used for control analysis. At first, active control of the system using a set of three thrusters is considered. Subsequently, a mixed control strategy involving the thrusters in conjunction with a length rate control law is used. The form of feedback is obtained from an appropriate analysis of the equations of motion; however, the final results are obtained numerically.

Description of the System and Its Dynamics

The system under consideration consists of a subsatellite of mass m_s supported by the shuttle through a tether having a mass per unit length ρ_t and instantaneous nominal length L (Fig. 1). The center of mass S of the system is assumed to coincide with that of the shuttle and moves in an elliptic orbit having an eccentricity e. The inclination of the orbital plane to the equatorial plane is given by an angle i.

The rotational motion of the tether is described by two angles α (pitch) and γ (roll), given in that order in and out of the orbital plane, respectively. These two rotations transform a set of orbital coordinate axes x_o, y_o, z_o (x_o along the orbit normal and y_o along the outward radial vector from the center of the Earth) to a new set of rotating axes x, y, z. At any instant, the y axis coincides with the line joining the centers of mass of the shuttle and subsatellite, i.e., along the rotated, undeformed tetherline. The elastic deformations of the tether are superposed on this line. The transverse deflections in x (out-of-plane) and z (in-plane) directions are denoted by u and w, respectively, while v is the longitudinal extension.

Using the extended Hamilton's principle, one can obtain both the partial differential equations governing the elastic displacements and the ordinary differential equations describing the rotational motion. They can be found in Ref. 6. For the purpose of analysis (numerical or otherwise), it is more convenient to discretize the partial differential equations. This can be done by using a Galerkin-type method. The transverse displacements are expanded as

$$u = L\sum_{i=1}^{n} A_i(t)\phi_i(y,t) \tag{1a}$$

$$w = L\sum_{i=1}^{n} B_i(t)\phi_i(y,t) \tag{1b}$$

and longitudinal extension as

$$v = L\sum_{i=1}^{p} C_i(t)\psi_i(y,t) \tag{1c}$$

where ϕ_i and ψ_i are two sets of admissible functions satisfying at least the geometric boundary conditions. In this paper, n and p are limited to 2, and the admissible functions are chosen as

$$\phi_i(y,t) = \sqrt{2}\sin(i\pi y/L), \qquad i = 1,2,\ldots \tag{2a}$$

$$\psi_i(y,t) = (y/L)^{2i-1}, \qquad i = 1,2,\ldots \tag{2b}$$

Corresponding discretized equations are given in the Appendix.

Generalized Forces Due to the Thrusters

Consider three thrusters placed on the subsatellite that can fire along local x, y, and z directions, respectively. The total thrust $\boldsymbol{T}$ acting on the subsatellite may be expressed as

$$\boldsymbol{T} = T_\gamma \boldsymbol{i} + T_c \boldsymbol{j} + T_\alpha \boldsymbol{k} \tag{3}$$

The rationale behind such a choice of subscripts will become clear a little later.

The position vector of the subsatellite relative to the center of mass of the system is

$$\begin{aligned}\boldsymbol{R}_s &= [L + v(L,t)]\,\boldsymbol{j} \\ &= [1 + C_1 + C_2]\,L\boldsymbol{j}\end{aligned} \tag{4}$$

It may be noted that $\boldsymbol{j}$ is a function of α and γ, and it can be shown that

$$\frac{\partial \boldsymbol{j}}{\partial \alpha} = c\gamma \boldsymbol{k} \tag{5a}$$

and

$$\frac{\partial \boldsymbol{j}}{\partial \gamma} = -\boldsymbol{i} \tag{5b}$$

Using the principle of virtual work, the generalized forces due to the thrust $\boldsymbol{T}$ are given by

$$Q_i = \boldsymbol{T}\cdot\frac{\partial \boldsymbol{R}_s}{\partial q_i}, \quad q_i \equiv \alpha, \gamma, A_i, B_i, C_i \tag{6}$$

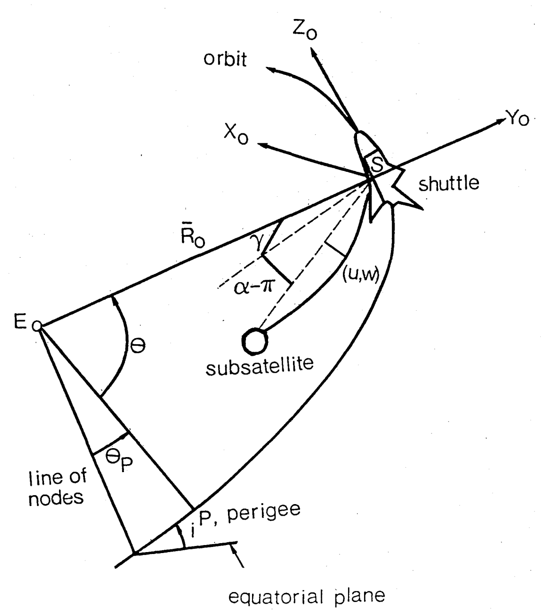


Fig. 1 Geometry of the system.

Using Eqs. (4-6), one obtains

$$Q_\alpha = T_\alpha L c\gamma(1 + C_1 + C_2) \tag{7a}$$

$$Q_\gamma = -T_\gamma L(1 + C_1 + C_2) \tag{7b}$$

$$Q_{c1} = Q_{c2} = T_c L \tag{7c}$$

$$Q_{A1} = Q_{A2} = Q_{B1} = Q_{B2} = 0 \tag{7d}$$

Choice of subscripts in Eq. (3) is now obvious in light of equations (7a-7c). The task of the x component of the thrust is to produce a torque to control out-of-plane rotational motion γ. The function of the y component of the thrust T_c is to control the longitudinal vibrations and to provide extra tension in the tether when its length is small. Finally, T_α acting along the z axis, i.e., approximately in the direction of flight, provides a torque to control in-plane rotation α. Note that no explicit generalized forces corresponding to transverse vibrational coordinates A_i and B_i are caused by the thrusters.

Equations (A1), (A2), and (A7) have been nondimensionalized through a division by $m_s L^2 \dot{\theta}^2$, where θ is the true anomaly. Thus, the corresponding nondimensional generalized forces S_α, S_γ, and S_{c1} (induced by the thrusters) are

$$S_\alpha = (1 + C_1 + C_2)\, T_\alpha c\,\gamma / (m_s L\dot{\theta}^2) \tag{8a}$$

$$S_\gamma = -(1 + C_1 + C_2) T_\gamma / (m_s L\dot{\theta}^2) \tag{8b}$$

$$S_{c1} = T_c / (m_s L\dot{\theta}^2) \tag{8c}$$

The modified C_2 equation (A8) has been obtained by subtracting the original C_2 equation from the C_1 equation (A7) for reasons associated with numerical convergence. Thus,

$$S_{c2} = 0 \tag{8d}$$

Defining

$$\tilde{T}_\alpha = T_\alpha / (m_s L\dot{\theta}^2),\ \tilde{T}_\gamma = T_\gamma / (m_s L\dot{\theta}^2),\ \tilde{T}_c = T_c / (m_s L\dot{\theta}^2) \tag{9}$$

Eqs. (8a-c) can be rewritten as

$$S_\alpha = (1 + C_1 + C_2)\tilde{T}_\alpha c\,\gamma \approx \tilde{T}_\alpha c\,\gamma \tag{10a}$$

$$S_\gamma = -(1 + C_1 + C_2)\tilde{T}_\gamma \approx -\tilde{T}_\gamma \tag{10b}$$

$$S_{c1} = \tilde{T}_c \approx \tilde{T}_c \tag{10c}$$

Derivation of Appropriate Functional Forms of Control Thrusts

It is not difficult to visualize that the rotational motion can be controlled by just requiring that $\tilde{T}_\alpha$ and $\tilde{T}_\gamma$ act along directions opposing the rotations α and γ at all times. However, it is not so obvious how these two thrust components can control the in-plane and out-of-plane transverse vibrations at the same time. Hence, an approximate analysis of the equations of motion is made *to obtain appropriate functional forms of control thrust only*.

Once the thrust expressions are determined, unsimplified Eqs. (A1-A8) are used for numerical results.

Simplified Equations

The following assumptions are made to obtain simplified equations that may give some information regarding the functional forms of thrusts required:

1) The orbit is assumed to be circular, and aerodynamic forces are ignored.

2) Equations of motion (A1-A8) are linearized.

3) C_2 is neglected compared to C_1.

4) Mass ratio $\nu = \rho_t L/m_s$ is assumed small compared to 1. This is valid at the most critical terminal phase of retrieval.

5) Some insignificant terms are neglected in the equations. For example, in Eq. (A3), $[3/2(\eta'' - F\eta') + (2 - \pi^2/3)\eta'^2]A_1$ is very small compared to $\pi^2\Omega^2 C_1 A_1$; vibratory terms have small influence on α equation (A1); etc.

With the above assumptions, one gets a set of simplified equations that may be put in the following form:

$$\begin{bmatrix} 1 & 0 & 0 \\ -k & 1 & 0 \\ \frac{1}{2}k & 0 & 1 \end{bmatrix} \begin{Bmatrix} \gamma'' \\ A_1'' \\ A_2'' \end{Bmatrix} + \begin{bmatrix} 2\eta' & 0 & 0 \\ -4k\eta' & 3\eta' & -8/3 \\ 0 & 8/3 & 3\eta' \end{bmatrix} \begin{Bmatrix} \gamma' \\ A_1' \\ A_2' \end{Bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ -4k & \omega_1^2 & 0 \\ 2k & 0 & 4\omega_1^2 \end{bmatrix} \begin{Bmatrix} \gamma \\ A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \\ 0 \end{Bmatrix} \tilde{T}_\gamma \tag{11a}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ k & 1 & 0 \\ -\frac{1}{2}k & 0 & 1 \end{bmatrix} \begin{Bmatrix} \alpha'' \\ B_1'' \\ B_2'' \end{Bmatrix} + \begin{bmatrix} 2\eta' & 0 & 0 \\ 4k\eta' & 3\eta' & -8/3 \\ 0 & 8/3 & 3\eta' \end{bmatrix} \begin{Bmatrix} \alpha' \\ B_1' \\ B_2' \end{Bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 3k & \omega_1^2 & 0 \\ -3k/2 & 0 & 4\omega_1^2 \end{bmatrix} \begin{Bmatrix} \alpha \\ B_1 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tilde{T}_\alpha - \begin{Bmatrix} 2 \\ 4k \\ 0 \end{Bmatrix} \eta' \tag{11b}$$

$$C_1'' + 2\eta' C_1' + \omega_c^2 C_1 = \tilde{T}_c + [3 - (\eta'' + \eta'^2)] \tag{11c}$$

where

$$k = \sqrt{2}/\pi = \text{constant}$$

$$\omega_1^2 = \pi^2 (EAC_1/\rho_t L^2 \dot{\theta}^2) = \pi^2 \Omega^2 C_1$$

$$\omega_c^2 = (EA/m_s L\dot{\theta}^2)$$

$$\eta = \ell n[L(\theta)/L(0)] = \text{a dimensionless length}$$

and E, A are the Young's modulus and area of cross section of the tether, respectively. Prime denotes differentiation with respect to true anomaly θ. It may be noted that for a tether having a constant length L, the fundamental frequency of transverse vibration is given approximately by ω_1 (where $C_1 = 3m_s L\omega^2/EA$) and that of longitudinal vibration is ω_c. The coupling of A_1 and A_2 with γ and that of B_1 and B_2 with α make it possible for $\tilde{T}_\gamma$ and $\tilde{T}_\alpha$ to control the transverse vibrations.

Derivation of Appropriate Form of $\tilde{T}_\gamma$

The simplified Eq. (11a) governing the out-of-plane motion is coupled very weakly to the in-plane motion (indirectly through ω). Hence, it can be investigated separately. Premultiplying the equation by the inverse of the matrix coefficient of the accelerations, one obtains

$$\begin{Bmatrix} \gamma'' \\ A_1'' \\ A_2'' \end{Bmatrix} + \begin{bmatrix} 2\eta' & 0 & 0 \\ -2k\eta' & 3\eta' & -8/3 \\ -k\eta' & 8/3 & 3\eta' \end{bmatrix} \begin{Bmatrix} \gamma' \\ A_1' \\ A_2' \end{Bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & \omega_1^2 & 0 \\ 0 & 0 & 4\omega_1^2 \end{bmatrix} \begin{Bmatrix} \gamma \\ A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} -1 \\ -k \\ k/2 \end{Bmatrix} \tilde{T}_\gamma \tag{12}$$

During monotonic retrieval of the subsatellite, η' is negative and the uncontrolled motion is unstable. In order to make γ, A_1, and A_2 stable, $\tilde{T}_\gamma$ must produce rate-dependent terms to overcome the negative damping. This suggests a form of $\tilde{T}_\gamma$,

$$\tilde{T}_\gamma = (-K_\gamma \gamma' - K_{A1} A_1' + K_{A2} A_2')\eta' \tag{13}$$

where K_γ, K_{A1}, and K_{A2} are positive numbers greater than 2, $3/k$, and $6/k$, respectively (note that $\eta' < 0$ during retrieval). The coefficients of A_1 and A_2 in Eq. (13) have different signs since the effects of $\tilde{T}_\gamma$ on the two motions as given in Eq. (12) have opposite signs.

Derivation of a Suitable Form of $\tilde{T}_\alpha$

Following an analysis similar to that for $\tilde{T}_\gamma$, one can get a simplified matrix equation for in-plane motion as

$$\begin{Bmatrix} \alpha'' \\ B_1'' \\ B_2'' \end{Bmatrix} + \begin{bmatrix} 2\eta' & 0 & 0 \\ 2k\eta' & 3\eta' & -8/3 \\ k\eta' & 8/3 & 3\eta' \end{bmatrix} \begin{Bmatrix} \alpha' \\ B_1' \\ B_2' \end{Bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & \omega_1^2 & 0 \\ 0 & 0 & 4\omega_1^2 \end{bmatrix} \begin{Bmatrix} \alpha \\ B_1 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -k \\ k/2 \end{Bmatrix} \tilde{T}_\alpha - \begin{Bmatrix} 2 \\ 2k \\ k \end{Bmatrix} \eta' \tag{14}$$

Through the coupling between pitch rotation and the vibrations, $\tilde{T}_\alpha$ can influence B_1 and B_2. Clearly, to make in-plane motion stable, $\tilde{T}_\alpha$ must depend on α', B_1', and B_2' as follows:

$$\tilde{T}_\alpha = (K_0 + K_\alpha \alpha' - K_{B1} B_1' + K_{B2} B_2')\eta' \tag{15}$$

where $K_0 = 2$ cancels the $2\eta'$ term on the right-hand side of the α equation and K_α, K_{B1}, and K_{B2} are positive numbers greater than 2, $3/k$, and $6/k$, respectively. The first term on the right-hand side of Eq. (15) suggests that $\tilde{T}_\alpha$ is likely to be much larger than $\tilde{T}_\gamma$ (assuming that a successful control system would make the motions small).

Functional Form of $\tilde{T}_c$

$\tilde{T}_c$ along the direction of the tetherline has two functions. One is to damp the longitudinal vibrations, i.e., control the unstable generalized coordinate C_1. The other is to provide a certain amount of tension in the tether during the terminal phase of retrieval.

This second function of $\tilde{T}_c$ is very important. Very small tension in the tether can lead to slackness in the tether. Theoretically, when L goes to zero, the tension in the tether approaches zero as well (except when the retrieval process is speeding up). Any longitudinal vibrations in that situation could make the tether slack.

The question arises as to how a minimum tension must be maintained in the tether. Obviously, the stronger the firing of the T_c thruster, the more tension is provided, thus making it more likely to prevent slackness in the tether. However, more fuel is consumed that way. Since the tension is fairly large when the tether is long, the second function of $\tilde{T}_c$ could be performed only at the terminal phase of retrieval.

Observing the C_1 equation (11c), it is not difficult to find an appropriate form of T_c to damp the unstable longitudinal vibrations—it simply has to be some negative multiple of C_1'. Keeping in mind that C_2 has been ignored in the simplified analysis, T_c is designated as

$$\tilde{T}_c = \begin{bmatrix} -K_c\omega_c(C_1' + C_2'), & L_i \ge L \ge L_T \\ -K_c\omega_c(C_1' + C_2') + 3(L_T - L)/L, & L < L_T \end{bmatrix} \tag{16}$$

such that $K_c\omega_c + 2\eta' > 0$ and L_T is a preset value of the length below which the thruster fires additionally to maintain a certain amount of tension. The term $3(L_T - L)/L$ in Eq. (16) causes the tension in the tether to remain approximately constant when L is reduced below L_T. The larger the value of length L_T, the larger the tension maintained in the tether; however, more fuel is then consumed. The choice of the value of L_T is limited by the maximum thrust provided by the thruster.

Numerical Results for Thruster Control

The functional forms of $\tilde{T}_\alpha$, $\tilde{T}_\gamma$, and $\tilde{T}_c$ have been obtained through an approximate analysis. To test the validity of these results, a computer simulation is carried out that uses the original lengthy equations of motion, i.e. Eqs. (A1-A8).

For numerical calculations, a spherical satellite with projected area 1 m^2 and mass 170 kg is considered. The orbit of the shuttle is assumed to be circular and polar with an orbital rate of $\omega = 1.1 \times 10^{-3}$ rad/s. The diameter and fully deployed length of the stainless steel tether are 0.325 mm and 100 km, respectively. The retrieval is exponential, i.e., $L' = L\tilde{c}$, with $\tilde{c} = c/\omega$ and $c = -4 \times 10^{-4}\,\mathrm{s}^{-1}$. Thus $\eta' = \tilde{c}$. The initial generalized displacements are $(\alpha - \pi) = 15°$, $\gamma = 1°$, $C_1 = 0.47 \times 10^{-2}$, $C_2 = -0.19 \times 10^{-3}$, $A_1 = 0.5 \times 10^{-4}$, $A_2 = -0.5 \times 10^{-4}$, $B_1 = -0.16 \times 10^{-2}$, and $B_2 = 0.48 \times 10^{-3}$, while the initial generalized velocities are all equal to zero. Only the aerodynamic forces are considered to obtain the environmental generalized forces.

Figure 2 shows the dynamical behavior of the system when the coefficients K_γ, etc., in Eqs. (13), (15), and (16) are chosen as follows:

$$K_\gamma = 2 + \nu + 2/(-\tilde{c}),\ K_{A1} = 10/(-\tilde{c}),\ K_{A2} = 20/(-\tilde{c})$$

$$K_0 = c^2\gamma(2+\nu),\ K_\alpha = c^2\gamma(2+\nu) + 2/(-\tilde{c}),$$

$$K_{B1} = K_{A1},\ K_{B2} = K_{A2}$$

$$K_c = 1,\ L_T = 3 \text{ km}$$

(Note that $(-\tilde{c}) = 0.36$ in the present case.)

The maximum thrust provided by any of the thrusters is limited to ± 5 N. Thus, if any of Eqs. (13), (15), and (16) yields a higher value of thrust, the equation is ignored and a 5-N (or -5-N) thrust is applied. Note that all the motions are well controlled during the retrieval process. For tether length less than 3 km, the tension is maintained at very low but nonzero levels (approximately 2 N, Fig. 2e). The ± 5-N limitation for the thrusters yielding T_c and T_α is clearly defined in Fig. 2d.

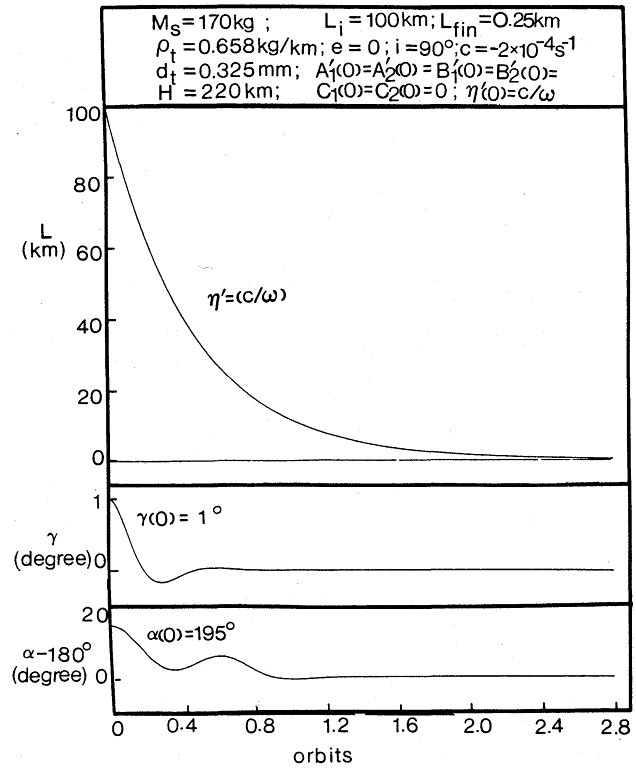
From Fig. 2d, it is estimated that the total thruster impulse required is about 500,000 Ns. It may be noted that the thruster providing T_α works hard at the beginning, consuming more thruster fuel than that providing T_c.

One may note that all the state variables are maintained at practically zero beyond $L = 5$ km. The requirements (small motion, etc.) for the validity of the simplified model [described by Eq. (11)] are then satisfied and numerical integration of the more complicated Eqs. (A1-A8) is no longer a necessity. However, the numerical integration was still continued until $L = 250$ m to verify that the same trend in the dynamical behavior persists. The time of retrieval from a length of 100 km to 250 m is approximately 4.2 h.

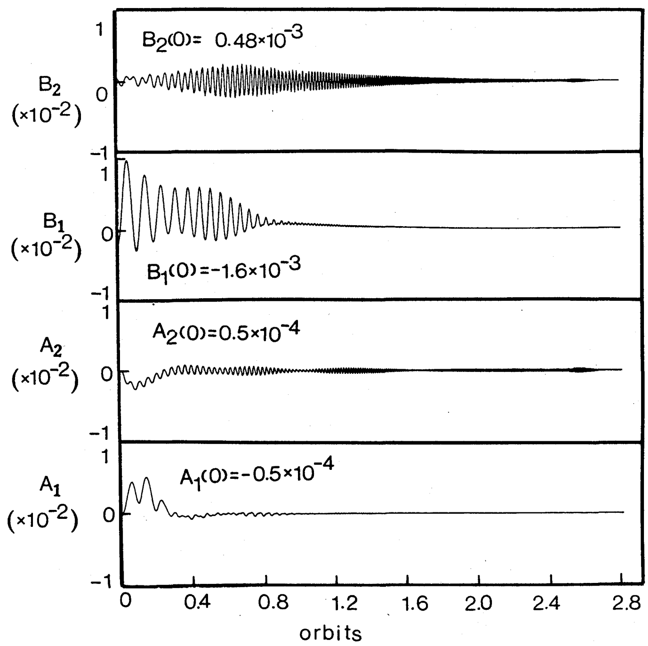
Although the results shown in Fig. 2 are satisfactory, they can be improved. There are two aspects: one is reduction of the thruster impulse; the other is reduction of the retrieval time. In the next section, a mixed control strategy is considered to attain these improvements.

A Mixed Control Strategy

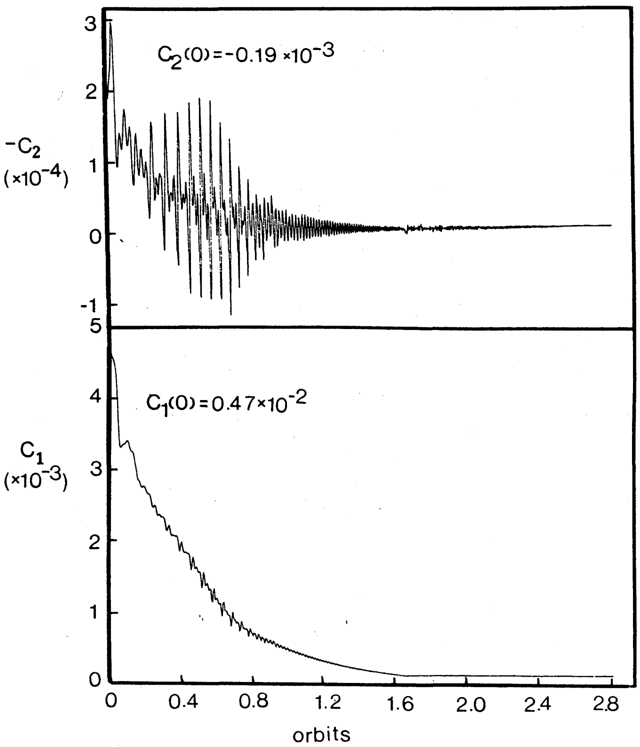
The thruster impulse required in the case shown in Fig. 2 is quite large. There are two reasons for this. One is that the



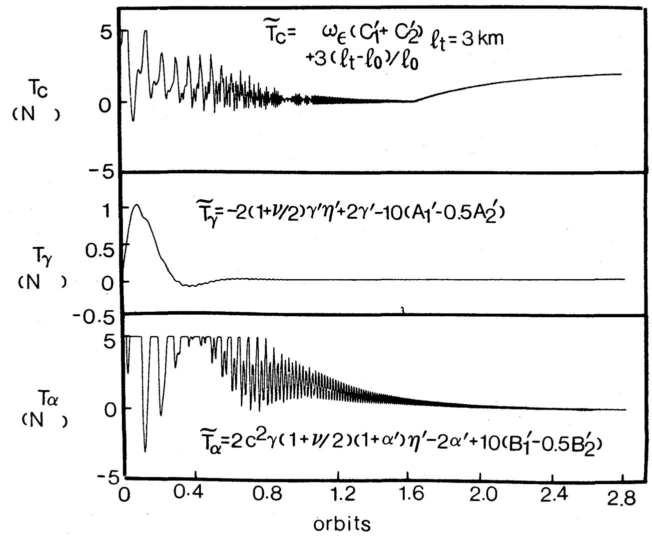
a) Variation of length and rotations.



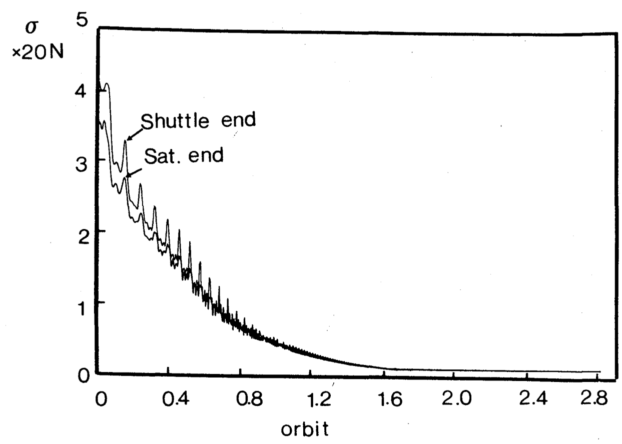
c) Behavior of transverse modal coordinates.



b) Behavior of longitudinal generalized coordinates.

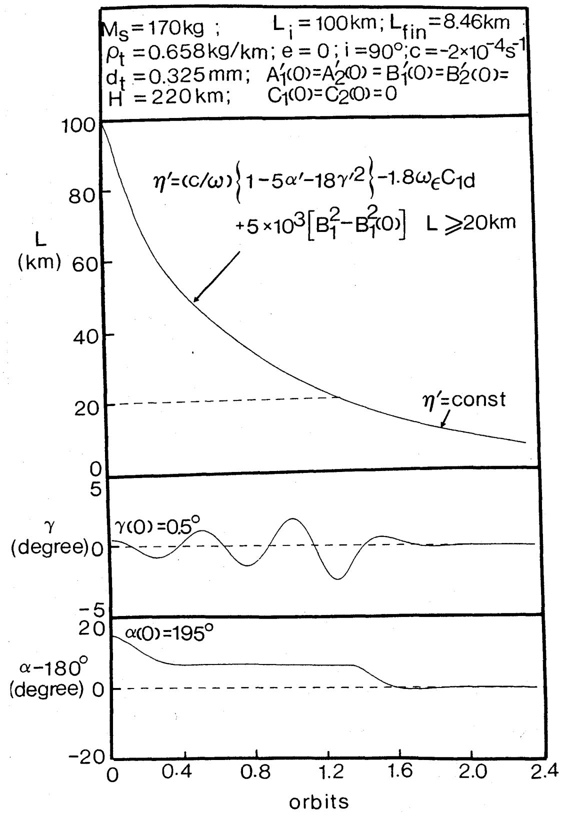


d) Time history of thrusts.

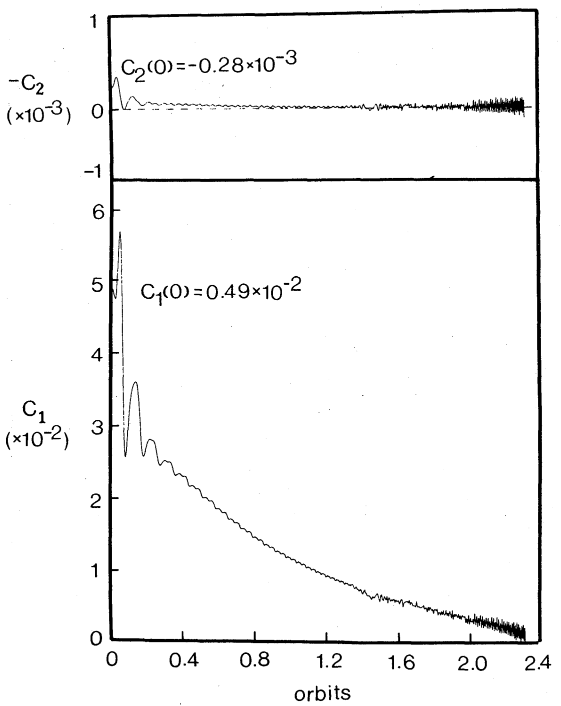


e) Variation of tension.

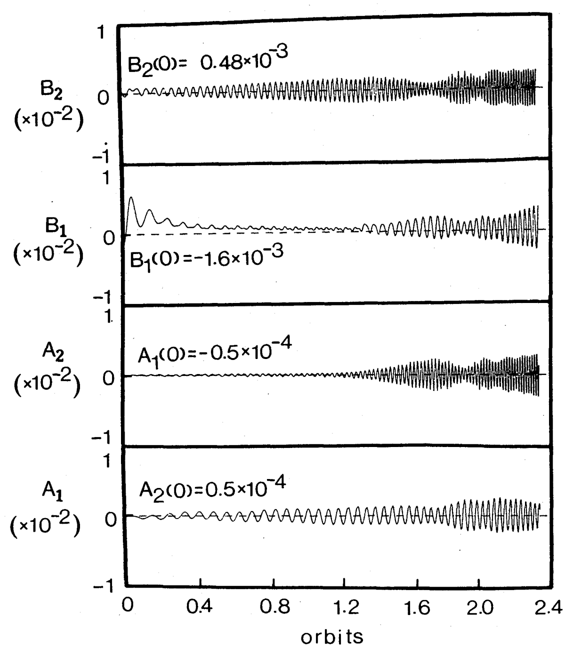
Fig. 2 Retrieval dynamics using solely thruster control.



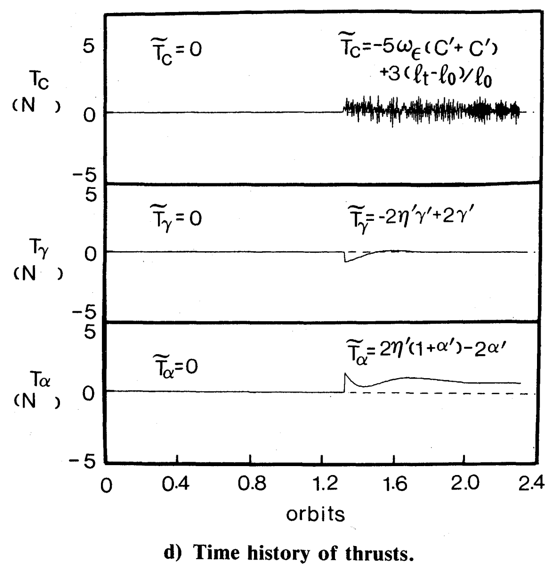
a) Variation of length and rotations.



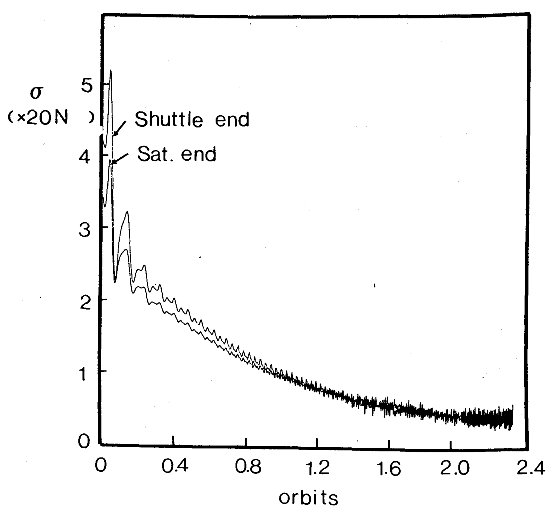
b) Behavior of longitudinal generalized coordinates.



c) Behavior of transverse modal coordinates.

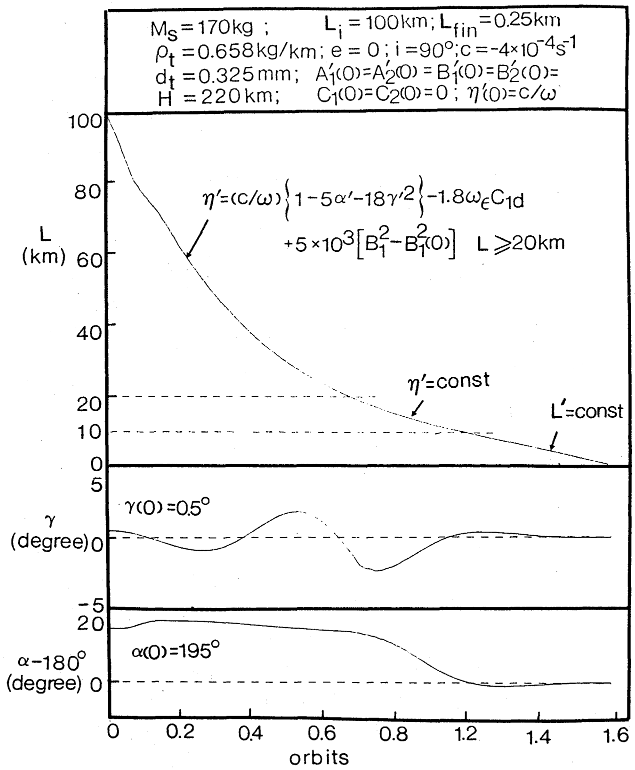


d) Time history of thrusts.

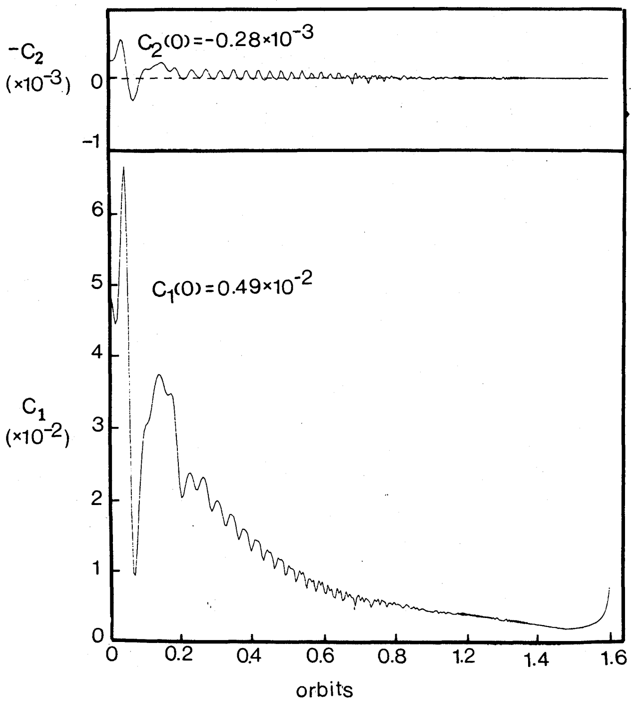


e) Variation of tension.

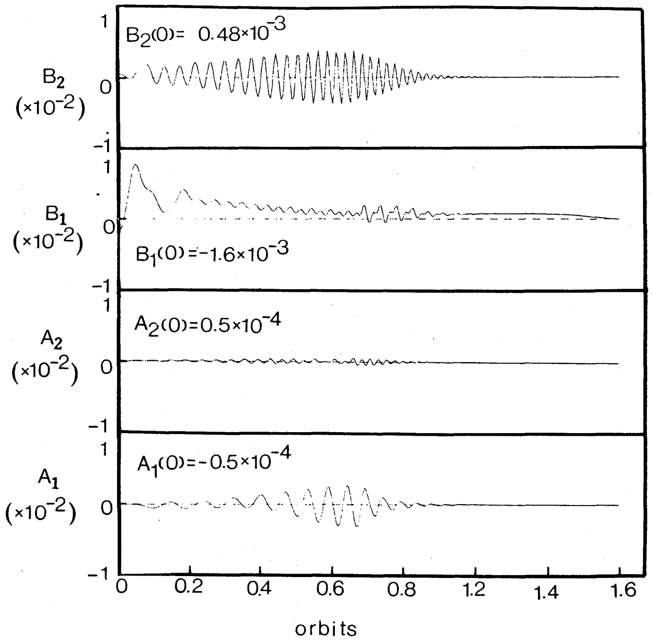
Fig. 3 Retrieval dynamics using mixed control (without feedback of transverse vibrations).



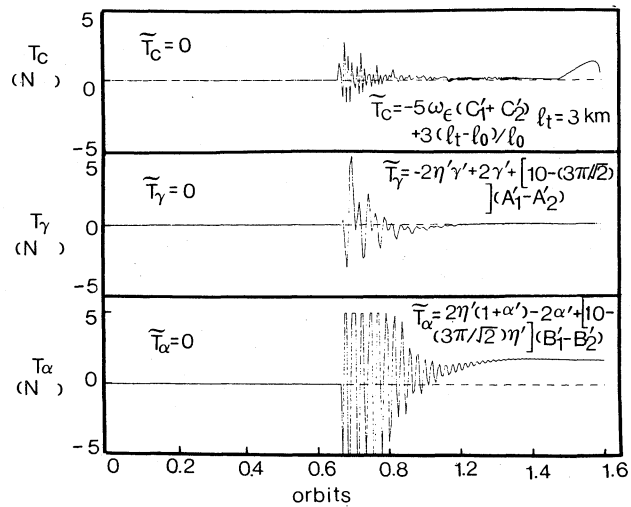
a) Variation of length and rotations.



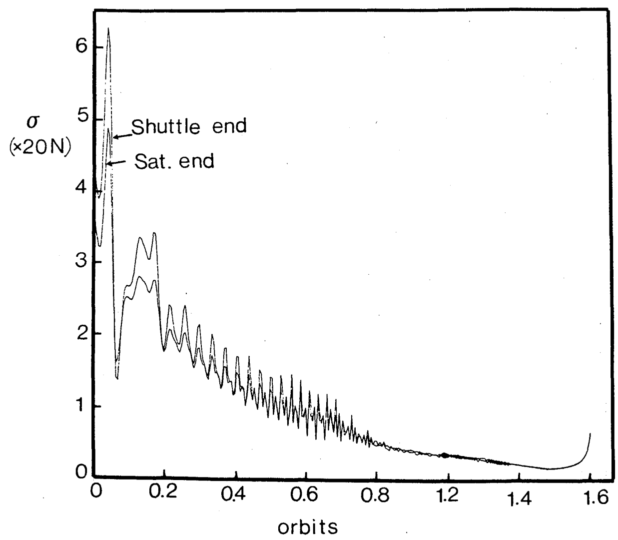
b) Behavior of longitudinal generalized coordinates.



c) Behavior of transverse modal coordinates.



d) Time history of thrusts.



e) Variation of tension.

Fig. 4 Retrieval dynamics using mixed control (with feedback of transverse vibrations).

thrusters fire from the beginning of the retrieval of the subsatellite and last for a long time. The second reason is that the retrieval is too slow at the terminal phase if an exponential retrieval is used all the time. The thrusters keep on firing to maintain certain minimum tension even though the retrieval process turns to a snail's pace. To overcome these difficulties, a mixed control strategy using thrusters as well as a length rate control law is proposed here.

The retrieval process is divided into two parts: one is from an initial length L_i to a length L_1, while the other is from L_1 to the end of retrieval. For the first part only a length rate law is used to control the motions; this is followed by a retrieval process using a thruster-augmented active control law to control the motions as well as to maintain a certain amount of tension in the tether. Below $L = L_2 (< L_1)$, a constant velocity retrieval is used instead of exponential retrieval to reduce the retrieval time.

Control Laws and Numerical Results for Mixed Control Strategy

The length rate law used for exponential retrieval from $L_i = 100$ km to $L_1 = 20$ km is

$$\eta' = \tilde{c}\{1 - \bar{K}_\alpha \alpha' - \bar{K}_\gamma \gamma'^2 - \bar{K}_c (C_1 - C_{1s}) - \bar{K}_{B1}[B_1^2 - B_1^2(0)]\} \tag{17}$$

where $\bar{K}_\alpha, \bar{K}_\gamma, \bar{K}_c, \bar{K}_{B1}$ are a set of gains, $C_{1s} = 3m_s L\omega^2/EA$ = (steady-state value of the tension corresponding to the instantaneous length)$/EA$, and $B_1(0)$ is the value of B_1 at the beginning of retrieval ($\theta = 0$). Note that η' given in Eq. (17) is simply a modulation of exponential retrieval $\eta' = \tilde{c} =$ a negative constant. In numerical calculations the following gains were used:

$$\bar{K}_\alpha = 1, \quad \bar{K}_\gamma = 9, \quad \bar{K}_c = 1.8\omega_c/(-\tilde{c}),$$
$$\bar{K}_{B1} = 5 \times 10^3/\tilde{c}, \quad \tilde{c} = -0.36$$

From $L = L_1 = 20$ km to $L = L_2 = 10$ km, an unmodulated exponential retrieval is used, i.e., all the gains in Eq. (17) are put to zero and the thrusters start firing. From 10 km onward, $\eta'' + \eta'^2 = 0$. This is equivalent to a constant retrieval rate L'. Care is taken so that η' (hence L') is continuous at $L = 20$ km and $L = 10$ km.

The physical parameters of the system considered in this section are the same as in the case of Fig. 2. The initial conditions are also identical except that $C_2 = -0.25 \times 10^{-3}$.

The importance of using the feedback of transverse vibrations is shown in Fig. 3. Here the thrusts are (for $L < 20$ km)

$$\tilde{T}_\alpha = 2\eta'(1 + \alpha') - 2\alpha' \tag{18a}$$

$$\tilde{T}_\gamma = -2\eta'\gamma' + 2\gamma' \tag{18b}$$

$$\tilde{T}_c = -5\omega_c(C_1' + C_2') + 3(L_T - L)/L, \quad L_T = 3 \text{ km} \tag{18c}$$

The transverse vibrations continue to grow (Fig. 3c) although the rotations approach zero (Fig. 3a). This causes the tension at both ends of the tether to oscillate rapidly (Fig. 3e). C_1 also is affected due to nonlinear coupling, and when L is about 6 km, C_1 becomes zero, which shows that the tether is slack somewhere along its length. The computer calculation is then terminated.

It may be mentioned here that if thrusters alone are used for control (i.e., no tension control or length rate control), slackening of the tether may not be a problem and the vibrations need not be controlled. One can then use thrusts as given in Eq. (18) from the beginning of the retrieval process itself. The rotations will be stable, the behavior being similar to (but not exactly equal to) that shown in Fig. 3a. This requires measurement of only the rates of α, γ, and $C_1 + C_2$; the last is related to the range measurement of the subsatellite. No vibration measurement is then needed.

The results shown in Fig. 4 use the length rate law, Eq. (17), until $L = 20$ km and then the thrust laws, Eqs. (13), (15), and (16), with feedback of transverse vibrations, where

$$K_\gamma = 2 + 2/(-\eta'), \quad K_{A1} = (3/k) + 10/(-\eta'), \quad K_{A2} = 2K_{A1}$$
$$K_0 = 2, \quad K_\alpha = 2 + 2/(-\eta'), \quad K_{B1} = K_{A1}, \quad K_{B2} = K_{A2}$$
$$K_c = 5, \quad L_T = 3 \text{ km}$$

Recall that $k = \sqrt{2}/\pi$ and that η' is a constant ($\tilde{c}$) during exponential retrieval but increases gradually in magnitude during constant velocity retrieval. Note that both $\alpha - \pi$ and γ go to zero (Fig. 4a); so do the transverse vibrations (Fig. 4c) and the longitudinal vibrational variable C_2 (Fig. 4b). C_1 is maintained at a certain finite value, as is the tension (Fig. 4e), during the terminal phase of retrieval. Thus, the overall dynamic response is quite satisfactory. The retrieval time is a little less than 1.5 orbits, i.e., about 2 h. The thruster impulse used is around 10,000 Ns, which is considerably smaller than the 50,000 Ns needed when thruster control alone is used.

Measurement of States Associated with Vibrations

Implementation of the control laws suggested above require the determination of A_1, A_2, B_1, B_2, the sum $C_1 + C_2$, and their rates. Discussion of the hardware requirement is beyond the scope of this paper; however, what physical quantities must be measured are specified below. It is proposed to carry out the following measurements:

1) Out-of-plane tether slopes (bowing angles) at the two ends (to determine A_1 and A_2): referring to Fig. 5, one may note that

$$\frac{\partial u}{\partial y}(0) = \tan\delta_1 \approx \delta_1, \quad \text{for small } \delta_1 \tag{19a}$$

$$\frac{\partial u}{\partial y}(L) = \tan(\pi - \delta_2) \approx -\delta_2, \quad \text{for small } \delta_2 \tag{19b}$$

Using Eqs. (1) and (19),

$$\sqrt{2}\pi(A_1 + 2A_2) = \delta_1, \qquad \sqrt{2}\pi(-A_1 + 2A_2) = -\delta_2$$

Hence,

$$A_1 = (2\sqrt{2}\pi)^{-1}(\delta_1 + \delta_2) \tag{20a}$$

and

$$A_2 = (4\sqrt{2}\pi)^{-1}(\delta_1 - \delta_2) \tag{20b}$$

2) In-plane tether bowing angles at the two ends (to determine B_1 and B_2): B_1 and B_2 are related to the in-plane slopes in a similar manner as in item 1.

3) Subsatellite range measurement (to determine $C_1 + C_2$): if R_s is the instantaneous distance between the shuttle and the subsatellite, then

$$C_1 + C_2 = (R_s/L) - 1$$

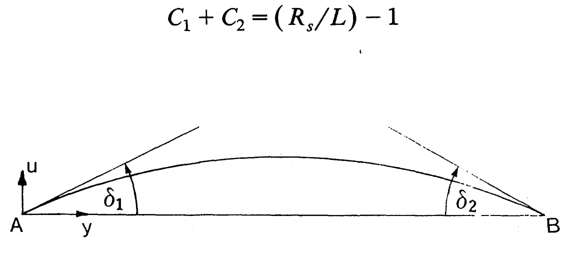


Fig. 5 Sketch to relate the slopes to in-plane vibrational coordinates.

Note that only the sum is needed in the control laws, not C_1 and C_2, separately.

Concluding Remarks

Control of the rotations and vibrations of a tethered subsatellite system during retrieval can be carried out successfully using a set of thrusters, acting alone or in conjunction with a length rate control law. The latter is preferable as it requires less thruster impulse. Functional forms of the thrusts are obtained in the paper through a simplified analysis and validated through numerical simulation of the original equations.

It appears that feedback of transverse vibrations is necessary if one wants to prevent slackness of the tether.

It is recommended that exponential retrieval be followed by uniform retrieval to reduce the time of retrieval and to conserve thruster fuel.

Appendix

Equations corresponding to the generalized coordinates $\alpha, \gamma, A_1, A_2, B_1, B_2, C_1$, and the modified equation corresponding to C_2 are given below.

$$
\begin{aligned}
&\{\alpha'' + (1+\alpha')[-F - 2\gamma'\,\mathrm{tg}\,\gamma + 2\eta'(1+\nu/2)/(1+\nu/3)] \\
&+3G\,\mathrm{s}\,\alpha\,\mathrm{c}\,\alpha\}\,\mathrm{c}^2\gamma(1+\nu/3) + k\nu\{[\alpha''\mathrm{s}(2\gamma) + (1+\alpha') \\
&\times(-F\,\mathrm{s}(2\gamma) + 3\eta'\mathrm{s}(2\gamma) + 2\gamma'\mathrm{c}(2\gamma))](A_1 - \tfrac{1}{2}A_2) \\
&+[(1+\alpha')\mathrm{s}(2\gamma)](A_1' - \tfrac{1}{2}A_2')]\} - 4k\nu\,\mathrm{c}\,\gamma\{(\eta'' - F\eta' \\
&+3\eta'^2 - \eta'\gamma'\,\mathrm{tg}\,\gamma)B_1 + \eta' B_1'\} + k\nu\,\mathrm{c}\,\gamma\{(B_1'' - \tfrac{1}{2}B_2'') \\
&+(6\eta' - F)(B_1' - \tfrac{1}{2}B_2') + [3(\eta'' - F\eta' + 3\eta'^2) \\
&+\gamma'^2 + (\gamma'' - F\gamma')\mathrm{tg}\,\gamma](B_1 - \tfrac{1}{2}B_2)\} \\
&+2(1+\alpha')\mathrm{c}^2\gamma\{(C_1' + C_2') + \eta'(C_1 + C_2)\} \\
&+2\nu\mathrm{c}^2\gamma\{(1+\alpha')(\tfrac{1}{3}C_1' + \tfrac{1}{5}C_2') + [\alpha'' + (1+\alpha') \\
&(-F - 2\gamma'\,\mathrm{tg}\,\gamma + 3\eta')](\tfrac{1}{3}C_1 + \tfrac{1}{5}C_2)\} = P_\alpha + S_\alpha \qquad \text{(A1)}
\end{aligned}
$$

$$
\begin{aligned}
&\{\gamma'' + \gamma'[-F + 2\eta'(1+\nu/2)/(1+\nu/3)] \\
&+[(1+\alpha')^2 + 3Gc^2\alpha]\,\mathrm{s}\,\gamma\,\mathrm{c}\,\gamma\}(1+\nu/3) \\
&+4k\nu\{(\eta'' - F\eta' + 3\eta'^2)A_1 + \eta' A_1'\} \\
&-k\nu\{(A_1'' - \tfrac{1}{2}A_2'') + (6\eta' - F)(A_1' - \tfrac{1}{2}A_2') \\
&+3(\eta'' - F\eta' + 3\eta'^2)(A_1 - \tfrac{1}{2}A_2)\} \\
&+k\nu\{2(1+\alpha')\mathrm{s}\,\gamma(B_1' - \tfrac{1}{2}B_2') \\
&+[\alpha''\mathrm{s}\,\gamma + (1+\alpha')(-F\,\mathrm{s}\,\gamma + \gamma'\mathrm{c}\,\gamma + 6\eta'\mathrm{s}\,\gamma)](B_1 - \tfrac{1}{2}B_2)\} \\
&+2\gamma'\{(C_1' + C_2') + \eta'(C_1 + C_2)\} \\
&+2\nu\{\gamma'(\tfrac{1}{3}C_1' + \tfrac{1}{5}C_2') + [\gamma'' + \gamma'(-F + 3\eta')] \\
&\times(\tfrac{1}{3}C_1 + \tfrac{1}{5}C_2)\} = P_\gamma + S_\gamma \qquad \text{(A2)}
\end{aligned}
$$

$$
\begin{aligned}
&A_1'' + (3\eta' - F)A_1' - \tfrac{8}{3}A_2' + \left[\tfrac{3}{2}(\eta'' - F\eta') + \left(2 - \frac{\pi 2}{3}\right)\eta'^2\right]A_1 \\
&-\tfrac{4}{3}(\eta'' - F\eta') + \tfrac{76}{9}\eta'^2]A_2 - k\{\gamma'' + \gamma'(-F + 4\eta') \\
&+[(1+\alpha')^2 + 3Gc^2\alpha]\,\mathrm{s}\,\gamma\,\mathrm{c}\,\gamma\} - 2k\gamma'\left[C_1' + \left(1 - \frac{6}{\pi^2}\right)\right. \\
&\times(C_2' - 2\eta' C_2)] + \pi^2\Omega^2\left\{\left[C_1 + \left(1 + \frac{3}{2\pi^2}\right)C_2\right]A_1\right. \\
&-\frac{20}{9\pi^2}C_2A_2 + \pi^2[\tfrac{3}{4}A_1^3 + \tfrac{3}{4}A_1B_1^2 + 6A_1A_2^2 \\
&+4A_2B_1B_2 + 2A_1B_2^2]\} - \{2(1+\alpha')\mathrm{s}\,\gamma B_1' \\
&+[\alpha''\mathrm{s}\,\gamma + (1+\alpha')(\gamma'\mathrm{c}\,\gamma - F\,\mathrm{s}\,\gamma + 3\eta'\mathrm{s}\,\gamma)]B_1 \\
&+[\tfrac{8}{3}(1+\alpha')\eta'\mathrm{s}\,\gamma]B_2\} = P_{A1} \qquad \text{(A3)}
\end{aligned}
$$

$$
\begin{aligned}
&A_2'' + (3\eta' - F)A_2' + \tfrac{8}{3}A_1' + [\tfrac{4}{3}(\eta'' - F\eta') - \tfrac{4}{9}\eta'^2]A_1 \\
&+\tfrac{3}{2}(\eta'' - F\eta') + \left(2 - \frac{4\pi^2}{3}\right)\eta'^2\Big]A_2 \\
&+\tfrac{1}{2}k\{\gamma'' - F\gamma' + [(1+\alpha')^2 + 3Gc^2\alpha]\,\mathrm{s}\,\gamma\,\mathrm{c}\,\gamma\} \\
&+k\gamma'\left[C_1' + \left(1 - \frac{3}{2\pi^2}\right)(C_2' - 2\eta' C_2)\right] \\
&+4\pi^2\Omega^2\left\{\left[C_1 + \left(1 + \frac{3}{8\pi^2}\right)C_2\right]A_2 - \frac{5}{9\pi^2}C_2A_1\right. \\
&\left.+\tfrac{1}{2}\pi^2[3A_1^2A_2 + 6A_2^3 + 2A_1B_1B_2 + A_2B_1^2 + 6A_2B_2^2]\right\} \\
&-\{2(1+\alpha')\mathrm{s}\,\gamma B_2' + [\alpha''\mathrm{s}\,\gamma + (1+\alpha') \\
&\times(\gamma'\mathrm{c}\,\gamma - F\,\mathrm{s}\,\gamma + 4\eta'\mathrm{s}\,\gamma)]B_2 \\
&-[\tfrac{8}{3}(1+\alpha')\eta'\mathrm{s}\,\gamma]B_1\} = P_{A2} \qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
&B_1'' + (3\eta' - F)B_1' - \tfrac{8}{3}B_2' + \left[\tfrac{3}{2}(\eta'' - F\eta') + \left(2 - \frac{\pi^2}{3}\right)\eta'^2\right]B_1 \\
&-[\tfrac{4}{3}(\eta'' - F\eta') + \tfrac{76}{9}\eta'^2]B_2 + k\,\mathrm{c}\,\gamma\{\alpha'' + (1+\alpha') \\
&\times(-F + 4\eta' - 2\gamma'\,\mathrm{tg}\,\gamma) + 3G\,\mathrm{s}\,\alpha\,\mathrm{c}\,\alpha\} + 2k(1+\alpha')\mathrm{c}\,\gamma \\
&\times\left[C_1' + \left(1 - \frac{6}{\pi^2}\right)(C_2' - 2\eta' C_2)\right] \\
&+\pi^2\Omega^2\left\{\left[C_1 + \left(1 + \frac{3}{2\pi^2}\right)C_2\right]B_1\right. \\
&\left.-\frac{20}{9\pi^2}C_2B_2 + \pi^2[\tfrac{3}{4}B_1^3 + \tfrac{3}{4}B_1A_1^2 + 4B_2A_1A_2 + 2B_1A_2^2]\right\} \\
&+\{2(1+\alpha')\mathrm{s}\,\gamma A_1' + [\alpha''\mathrm{s}\,\gamma + (1+\alpha') \\
&\times(\gamma'\mathrm{c}\,\gamma - F\,\mathrm{s}\,\gamma + 3\eta'\mathrm{s}\,\gamma)]A_1 \\
&+[\tfrac{8}{3}(1+\alpha')\eta'\mathrm{s}\,\gamma]A_2\} = P_{B1} \qquad \text{(A5)}
\end{aligned}
$$

$$B_2'' + (3\eta' - F)B_2' + \tfrac{8}{3}B_1' + \left[\tfrac{4}{3}(\eta'' - \eta' F) - \tfrac{4}{9}\eta'^2\right]B_1$$
$$+ \left[\tfrac{3}{2}(\eta'' - F\eta') + (2 - \tfrac{4}{3}\pi^2)\eta'^2\right]B_2 - \tfrac{1}{2}k\,\mathrm{c}\,\gamma\{\alpha'' + (1+\alpha')$$
$$\times(-F - 2\gamma'\,\mathrm{tg}\,\gamma) + 3G\,\mathrm{s}\,\alpha\,\mathrm{c}\,\alpha\}$$
$$-k(1+\alpha')\mathrm{c}\,\gamma\left[C_1' + \left(1 - \frac{3}{2\pi^2}\right)(C_2' - 2\eta' C_2)\right]$$
$$+4\pi^2\Omega^2\left\{\left[C_1 + \left(1 + \frac{3}{8\pi^2}\right)C_2\right]B_2 - \frac{5}{9\pi^2}C_2 B_1\right.$$
$$\left. + \frac{\pi^2}{2}\left[3B_1^2 B_2 + 6B_2^3 + 2B_1 A_1 A_2 + A_1^2 B_2 + 6A_2^2 B_2\right]\right\}$$
$$+\{2(1+\alpha')\mathrm{s}\,\gamma A_2' + [\alpha''\mathrm{s}\,\gamma + (1+\alpha')$$
$$\times(\gamma'\mathrm{c}\,\gamma - F\,\mathrm{s}\,\gamma + 4\eta'\mathrm{s}\,\gamma)]A_2$$
$$-[\tfrac{8}{3}(1+\alpha')\eta'\mathrm{s}\,\gamma]A_1\} = P_{B2} \tag{A6}$$

$$(1+\tfrac{1}{3}\nu)(C_1'' - FC_1') + (1+\tfrac{1}{5}\nu)(C_2'' - FC_2') + (2+\nu)\eta' C_1'$$
$$+(2+\tfrac{7}{10}\nu)\eta' C_2' + (1 + \tfrac{1}{2}\nu + C_1 + C_2)(\eta'' + \eta'^2 - F\eta')$$
$$-(1+\tfrac{1}{3}\nu)\{(1+\alpha')^2 c^2\gamma + \gamma'^2 + (3c^2\alpha c^2\gamma - 1)G\}$$
$$+\omega_c^2\left[C_1 + C_2 + \frac{\pi^2}{2}(A_1^2 + 4A_2^2 + B_1^2 + 4B_2^2)\right]$$
$$+2k\nu\{\gamma'(A_1' - \tfrac{1}{2}A_2') + [\gamma'' + \gamma'(-F + 3\eta')](A_1 - \tfrac{1}{2}A_2)\}$$
$$-2k\nu\,\mathrm{c}\,\gamma\{(1+\alpha')(B_1' - \tfrac{1}{2}B_2')$$
$$+[\alpha'' + (1+\alpha')(-F + 3\eta' - \gamma'\mathrm{tg}\,\gamma)]$$
$$\times(B_1 - \tfrac{1}{2}B_2)\} = P_{c1} + S_{c1} \tag{A7}$$

$$\tfrac{2}{15}(C_1'' - FC_1') + \tfrac{2}{35}(C_2'' - FC_2') + \tfrac{1}{2}\eta' C_1' + \tfrac{19}{70}\eta' C_2'$$
$$+\tfrac{1}{2}(\eta'' + \eta'^2 - F\eta') - \tfrac{2}{15}\{(1+\alpha')^2 c^2\gamma + \gamma'^2$$
$$+(3c^2\alpha c^2\gamma - 1)G\} + \Omega^2\{-\tfrac{4}{5}C_2 - \tfrac{3}{4}(A_1^2 + A_2^2 + B_1^2 + B_2^2)$$
$$+\tfrac{40}{3}(A_1A_2 + B_1B_2)\} + \frac{12}{\pi^2}k\{\gamma'(A_1' - \tfrac{1}{8}A_2')$$
$$+[\gamma'' + \gamma'(-F + 3\eta')](A_1 - \tfrac{1}{8}A_2)\} - \frac{12}{\pi^2}k\,\mathrm{c}\,\gamma\{(1+\alpha')$$
$$\times(B_1' - \tfrac{1}{8}B_2') + [\alpha'' + (1+\alpha')(-F - \gamma'\mathrm{tg}\,\gamma + 3\eta')]$$
$$\times(B_1 - \tfrac{1}{8}B_2)\} + 4k\eta'\left(1 - \frac{6}{\pi^2}\right)[(1+\alpha')\mathrm{c}\,\gamma B_1 - \gamma' A_1]$$
$$-2k\eta'\left(1 - \frac{3}{2\pi^2}\right)[(1+\alpha')\mathrm{c}\,\gamma B_2 - \gamma' A_2]$$
$$= (P_{c2} + S_{c2})_{\mathrm{mod}} \tag{A8}$$

In the above equations,

$$\nu = \rho_t L/m_s, \qquad k = \sqrt{2}/\pi, \qquad \eta = \ell n(L/L_{\mathrm{ref}})$$
$$F = 2es\theta/(1 + ec\theta), \qquad G = 1/(1 + ec\theta)$$
$$\Omega^2 = EA/\rho_t L^2\theta^2, \qquad \omega_c^2 = EA/m_s L\theta^2$$

Sin and cos are abbreviated as s and c, respectively; P_i and S_i are generalized environmental and control forces, respectively.

Acknowledgments

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